SU_3 Symmetry and the Existence of a Ninth Vector Meson*

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(Received 18 March 1963)

Baryon-baryon and baryon-antibaryon interactions are discussed within the framework of the SU_3-symmetric octet model without assuming R invariance. Although this model, at least in the present version, contains a free parameter in the specification of the baryon–vector meson coupling, it turns out that the predictions of the model are not consistent with the experimental determinations of the baryon–vector meson coupling constants, if only exchanges of members of the vector meson octet are considered. As the simplest way of removing such discrepancy, the presence of a ninth vector meson is proposed, a unitary singlet, for which some experimental support is available.

I. INTRODUCTION

THE aim of the present paper is to investigate the extent to which the values of the baryon–vector meson coupling constants, as calculated from the octet model of the SU_3-symmetry scheme,[1] can be reconciled with the known empirical facts regarding baryon-baryon and baryon-antibaryon interactions. In the octet model the eight vector mesons, as well as the eight pseudoscalar mesons, may be thought of as bound states of a baryon-antibaryon pair or, strictly speaking, as superpositions of such states. The probability for a virtual decay of some meson into a particular baryon-antibaryon pair has the form of a product of a coupling constant, characteristic of the particular members of the baryon and meson octet in question, times a factor describing the general dependence on spin and energy. The coupling constant may again be factorized in a universal baryon–vector meson (or baryon–pseudoscalar meson) coupling constant f and a coefficient G expressing the dependence on the generalized isobaric spin.

It is well known[1] that a certain ambiguity exists in the choice of coupling between three fields each of which belongs to the eight-dimensional representation of SU_3. In fact, the circumstance that the decomposition of the Kronecker product 8×8 contains the eight-dimensional representation itself twice, allows the construction of two orthogonal eigenstates of the Hamiltonian corresponding to the same spatial quantum numbers and the same values of the isospin T, T_3 and the hypercharge Y; the degeneracy being a consequence of the invariance of the Hamiltonian with respect to a permutation of the particles involved. Thus, from two single-particle states $|a\rangle$ and $|b\rangle$, each of which is a carrier of the eight-dimensional representation, it is possible to form either an antisymmetrical or a symmetrical two-particle state $|c\rangle$:

$$|c\rangle_A=F_{ab}{}^c|a\rangle|b\rangle,$$
$$|c\rangle_S=D_{ab}{}^c|a\rangle|b\rangle. \quad (1)$$

Each index a, b, and c runs from 1 to 8 and the quantities $F_{ab}{}^c$ are totally antisymmetrical in all three indexes, whereas the coefficients $D_{ab}{}^c$ are completely symmetrical.

Although in certain cases, where at least two of the particles involved are identical, and thus Bose or Fermi statistics may be invoked, it is possible to make an unambiguous choice between the F and the D coupling; such uniqueness in the specification of the coupling requires, in general, the additional assumption of invariance of the interaction under the R-reflection operation.[1,2] More recent investigations seem, however, to raise rather large difficulties for the hypothesis of R-invariant interactions. In particular, it has been pointed out by Cutkosky[3] that with the assumption of R invariance the prediction of the existence of another baryon octet appears to be unavoidable. In the general case, when no assumption of R invariance is made, an additional parameter, describing the "F-D mixing ratio," enters for the characterization of the octet couplings. Indeed, the $B\bar{B}V$ vertex has the following dependence on the unitary spin-coupling matrices:

$$f\cdot G_{\alpha\beta}{}^r=(aF_{\alpha\beta}{}^r+bD_{\alpha\beta}{}^r)\cdot f. \quad (3)$$

Here α, β, and r refer to the in- and outgoing baryon and the vector meson, respectively, whereas f denotes the universal BV-coupling constant. The F-D mixing ratio need, of course, not be the same for the baryon-pseudoscalar coupling. The normalization of the matrices F and D is fixed by the following introduction of the "mixing angle" ψ:

$$a=(1/18)^{1/2}\sin\psi,$$
$$b=(1/10)^{1/2}\cos\psi. \quad (4)$$

II. THE EXISTENCE OF A HARD CORE IN THE BARYON-BARYON POTENTIAL

The low binding energy of the deuteron and the change of sign of the S-wave phase shifts in nucleon-nucleon scattering is generally traced to the presence of a repulsive core in the nucleon-nucleon potential. The experimental values available for the binding

* Supported by the U. S. Atomic Energy Commission.
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[1] M. Gell-Mann, California Institute of Technology Report CTSL-20 (unpublished); Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961). (Further references to basic papers on SU_3 symmetry are listed in Refs. 3 and 8.)
[2] M. Gell-Mann and S. L. Glashow, Ann. Phys. (N. Y.) **15**, 437 (1961).
[3] R. E. Cutkosky (to be published).

TABLE I. Eigenvalues of the operator $\mathfrak{M}$. $[a/b=(\frac{1}{3}\sqrt{5})\tan\psi]$.

d	State $(\lambda\mu)$	$\mathfrak{M}$
1	$(00)_S$	$-12a^2+(20/3)b^2=\frac{2}{3}\cos^2\psi$
8	$(11)_A$	$-6a^2+(10/3)b^2=\frac{1}{3}\cos^2\psi$
8	$(11)_S$	$-6a^2-2b^2=-(\frac{1}{3}\sin^2\psi+\frac{1}{5}\cos^2\psi)$
10	$(30)_A$	$-(8/3)b^2+8ab=-(4/15)\cos^2\psi+(4/\sqrt{45})\sin\psi\cos\psi$
10	$(03)_A$	$-(8/3)b^2-8ab=-(4/15)\cos^2\psi-(4/\sqrt{45})\sin\psi\cos\psi$
27	$(22)_S$	$4a^2+\frac{4}{3}b^2=\frac{2}{3}(\frac{1}{3}\sin^2\psi+\frac{1}{5}\cos^2\psi)$

energies of the hyperfragments indicate that the idea of a repulsive core may apply to all baryon-baryon interactions.[4,5] Indeed, a rough comparison with the ground state of the hydrogen atom, taking appropriate values for the ratios between the coupling constants and masses involved, leads us, for the hyperfragments, to expect binding energies of hundreds of MeV. In contrast, the ${}_{\Lambda}H^2$ system is presumably not bound at all, and the binding energy for the ${}_{\Lambda}H^3$ fragment is around 0.2 MeV. Similarly, although the absence of Σ fragments may be explained as a result of a conversion of the Σ into a Λ in presence of neutrons, it is hard to understand why only a single Ξ fragment has been seen if these systems were reasonably strongly bound. (Wilkinson[6] reports the finding of a ${}_{\Xi}He^8$ with a binding energy about 6 MeV.)

It was early suggested that the presence of a repulsive short-range potential could be explained as a result of the exchange of a vector meson of appropriate mass between the two interacting baryons. In fact, the simple graph shown in Fig. 1 gives rise to a potential of the type[7]

$$V=f^2\,(1/r)e^{-Mr}+\text{spin-dependent terms}, \qquad (5)$$

where M is the mass of the exchanged vector meson.

In case the exchanged vector meson is coupled to the isospin and hypercharge current, the sign of the central potential will, of course, in general, depend on the value of the isospin and hypercharge for the two-baryon state in question. In particular, within the framework of SU_3 symmetry, the signs of the potential in the various unitarity spin states $|\lambda\mu\rangle$, corresponding

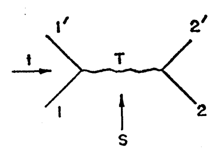

FIG. 1. Vector meson exchange diagram.

[4] H. P. Noyes, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissons (Interscience Publishers, Inc., New York, 1960); H. Stapp, M. Moravcsik, and H. P. Noyes, *ibid.*; G. Breit, *ibid.*

[5] R. D. Dalitz, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at Cern, Geneva* (CERN, Geneva, 1962).

[6] D. H. Wilkinson, Phys. Rev. Letters **3**, 397 (1959).

[7] R. S. McKean, Phys. Rev. **125**, 1391 (1962); P. T. Matthews, in Proceedings of the Aix en Provence International Conference on Elementary Particles, 1962 (unpublished).

to exchange of a member of the vector meson octet, can be evaluated. In the symbols of Eq. (3), the generalized isobaric-spin dependence of the diagram in Fig. 1 is given by

$$\mathfrak{M}_{ac,bd}=G_{ab}{}^{r}G_{cd}{}^{r}$$
$$=a^2M_{ac,bd}+b^2\bar{M}_{ac,bd}+2ab[W_{ac,bd}]_{10}, \qquad (6)$$

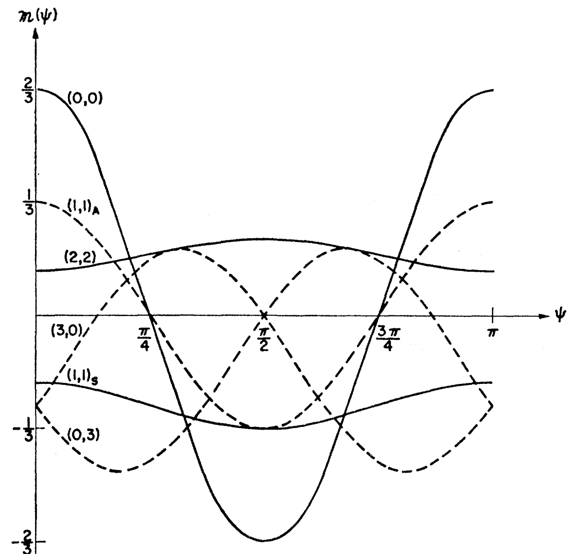

FIG. 2. $B\bar{B}$ potentials in the different states $(\lambda\mu)$, plotted as functions of the mixing angle ψ.

where M, $\bar{M}$, and W are the combinations of matrices employed by Cutkosky *et al.*[8] They are given by

$$M=F^rF^r,\quad \bar{M}=D^rD^r,\quad W=F^rD^r. \qquad (7)$$

The square bracket with subscript 10 around W in Eq. (6) indicates that this term only contributes in the states (30) and (03). The eigenvalues of the operators M, $\bar{M}$, and $\bar{W}$ in the various states $(\lambda\mu)$ are given by Tarjanne.[9] The corresponding eigenvalues of $\mathfrak{M}$ are listed in Table I. The potentials are plotted as functions of ψ in Fig. 2 in the $(\lambda\mu)$ representation. This representation corresponds to scattering states which do not contain baryons of a well-defined type. The situation

[8] R. E. Cutkosky, J. Kalckar, and P. Tarjanne, Phys. Letters **1**, 93 (1962).

[9] P. Tarjanne, Ann. Acad. Sci. Fennicae Ser. A VI, **105** (1962).

is, in fact, somewhat similar to that met with in the Paschen-Back effect in atomic spectra representing the transition from the "weak coupling" (or j-j coupling) to the strong coupling (or LS coupling) limit. Thus, as long as the baryons are so far apart from each other that they may be considered as free particles, we may characterize the state by the individual isospin and hypercharge quantum numbers t_1, t_{1z}, t_2, t_{2z}, Y_1, and Y_2 of the baryons. In contrast, when the particles enter into the region of strong interaction, which is nondiagonal in the individual quantum numbers of the particles, the state is characterized exclusively by the conserved quantities $T=|\mathbf{t}_1+\mathbf{t}_2|$, $T_z=t_{1z}+t_{2z}$, $Y=Y_1+Y_2$, and $(\lambda\mu)\subset(11)\otimes(11)$, and it is built up by superposition of states containing two baryons of specified type.

If we consider the scattering of two such particles of a definite type, say N and Λ, originally far apart from each other, we may, therefore, conclude from the experimental observations that no strong attraction can exist in any of the states $|\lambda\mu\rangle$ which can be formed from the two particle system in question. However, the $N\Lambda$ state is characterized by the quantum numbers $T=\frac{1}{2}$, $Y=1$ and, thus, contains nonvanishing components of the states $|22\rangle$, $|(11)_A\rangle$, $|(11)_S\rangle$, and $|03\rangle$, of which the state $|(11)_S\rangle$ exhibits an attractive core for all values of ψ. For NN scattering ($Y=2$), the $T=0$ state is a pure (03) state, whereas the $T=1$ state is identified as the (22) state. For pp scattering we have, thus, in particular,

$$\mathfrak{M}_{pp}(\psi)=\tfrac{2}{3}(\tfrac{1}{3}\sin^2\psi+\tfrac{1}{5}\cos^2\psi)\,. \tag{8}$$

Since this quantity is positive definite, we see that SU_3 symmetry, for any value of ψ, reproduces the repulsive core in pp scattering. In general, the distribution of attraction and repulsion among the various states $|\lambda\mu\rangle$ will of course depend on the value of the mixing angle ψ. However, as shown in the Appendix of Ref. 3, and also directly checked from Table I, the weighted mean value over all representations $(\lambda\mu)$ of the potential $\mathfrak{M}$ vanishes, i.e.,

$$\textstyle\sum_R d_R\mathfrak{M}(R)=0\,, \tag{9}$$

where d_R denotes the dimension of the representation R. Hence, it follows that the ratio between the total amount of attraction and the total amount of repulsion is fixed. All we achieve by varying ψ is to shift the attractive core from some states to others.

The simplest way of attaining the necessary repulsion in all states is by postulating the existence of a ninth vector meson, a unitary singlet. Being a singlet, this vector meson would mediate the same amount of repulsion in all BB states. Since the decomposition of the Kronecker product 8×8 includes the one-dimensional representation, it is possible within the scheme of SU_3 symmetry to describe the singlet as a baryon-antibaryon state in analogy with the octet mesons. Such possibility rests on the assumption that a strong

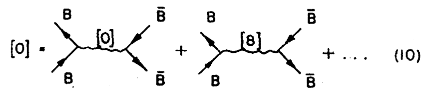

FIG. 3. Symbolic expansion of [0].

attraction exists in the (0,0) state of the $B\bar{B}$ system. This attraction is partly provided by the singlet itself and partly by the octet, i.e., we may symbolically write [0] in the form given in Fig. 3. The coupling constant $g_{B[0]}$ is, of course, quite independent of the coupling constant $g_{B[8]}$ as far as SU_3 symmetry is concerned. We shall later return to the possibilities of estimating the ratio $g_{B[0]}/g_{B[8]}$. In the present context it suffices to say that in order to compensate for the strong attraction we may presumably have that

$$g_{B[0]}\gg g_{B[8]}\,. \tag{10}$$

In that case, the second graph in Fig. 3 may cause merely a small perturbation of the mass spectrum as derived from the singlet exchange graph alone. This implies that we, along with the singlet itself, would encounter 63 other degenerate vector meson states. Clearly, this means that the exchange of other multiplets must contribute in an important manner to the structure of the singlet, and the whole question becomes a matter of examining the self-consistency equations for the vector meson multiplets. We hope to be able to return to this problem in a future publication, and restrict ourselves here to remembering that it might well happen that the Regge trajectories associated with such a hypothetical multiplet, under influence of the symmetry-breaking forces (which are rather strong after all), bend over without ever reaching the value $j=1$. In that case this multiplet would not turn up as actual resonances, although a particularly strong baryon-baryon interaction would be expected at energies where a member of this family of trajectories approached the value $j=1$.

Recently Sakurai[10] has directed attention to a report by the Brookhaven-Syracuse group who have found some evidence for the existence of a narrow resonance ($\Gamma<20$ MeV) in the $K\bar{K}$ system, with a mass of about 1020 MeV. If the spin-isospin assignment of this resonance turns out to be 1, 0, we have here a ninth vector meson, which we shall refer to as φ. The ω meson is usually considered as the $T=0$ member of the unitary octet. In that case the φ is to be identified with the unitary singlet. The $K\bar{K}$ decay of the φ will then be forbidden by SU_3 invariance. Alternatively, the φ may be regarded as the $T=0$ member of the octet, thus identifying the ω with the unitary singlet. In his paper, Sakurai points out that since the ω and the φ have in common the values of all the usual quantum numbers like spin, parity, isospin, G parity, and hypercharge, a particularly strong mixing of the SU_3-

[10] J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962).

invariant states is to be expected. In the very crude approximation where the interference with all the other vector states is neglected, one may write

$$|\varphi\rangle=\lambda_1|[0]\rangle-\lambda_2|[8],T=0\rangle,\quad |\omega\rangle=\lambda_2|[0]\rangle+\lambda_1|[8],T=0\rangle,\quad \lambda_1^2+\lambda_2^2=1. \tag{11}$$

A similar relation holds for the strength of the φB and the ωB couplings and we get in virtue of Eq. (10):

$$f=\lambda_1 g_0-\lambda_2 g_1\approx\lambda_1 g_0,\quad g=\lambda_2 g_0+\lambda_1 g_1\approx\lambda_2 g_0, \tag{12}$$

where

$$f=g_{B\varphi},\quad g=g_{B\omega},\quad g_0=g_{B[0]},\quad \text{and}\quad g_1=g_{B[8]_{T=0}}.$$

It is known that the Okubo mass formula,[11]

$$m=m_0\{1+\alpha Y+\beta[T(T+1)-\tfrac{1}{4}Y^2]\}, \tag{13}$$

which works so remarkably well for the baryon and pseudoscalar meson octets, predicts the mass of the $T=0$ member of the vector meson octet to be 930 MeV when the parameters α and β are adapted to the values 750 and 885 MeV for the ρ mass and M mass, respectively.[12] Following Sakurai we now assume that in the absence of the ω-φ mixing, the mass of the $T=0$ member of the octet should be given exactly by the formula (13).

We have already seen that the octet and the singlet are expected to be nearly degenerate in the absence of symmetry-violating interactions. With this assumption, it is possible from the actual values 1020 and 787 MeV for the masses of the φ and the ω, respectively, to estimate the coefficients λ_1 and λ_2 of Eq. (11) as the eigenvectors of the 2×2 potential matrix. One obtains in this way

$$\lambda_1=0.63,\quad \lambda_2=0.78, \tag{14}$$

showing that the φ contains a larger amount of the octet state, whereas the ω is mainly the singlet state. For this reason we adopt the notation ω_0 for the singlet and φ_0 for the $T=0$ member of the octet. We see, indeed, that the admixture is rather strong. From (12) and (14) we get the ratio between the effective coupling constants:

$$g^2/f^2=g_{B\omega}^2/g_{B\varphi}^2=1.56. \tag{15}$$

It need hardly be emphasized that this estimate is not to be taken too literally.

Attempts of fitting the isoscalar part of the nucleon form factors with an expression of the type

$$F_i^{(S)}=\frac{\lambda_2 g_0\gamma}{M_\omega^2-t}+\frac{\lambda_1 g_0\gamma}{M_\varphi^2-t}+c_i^{(S)} \tag{16}$$

have turned out to be inconclusive due to the large experimental uncertainties in the determination of the form factors. In fact, it seems at present that two poles are needed for the fitting of the data for the isovector as well as the isoscalar part of the nucleon form factor. Therefore, one can hardly attach any deeper significance to the circumstance that the mass of the second pole in both cases comes out more or less equal to that of the φ meson. Presumably the form factors can be fitted about as well for any position of the second pole between 1000 and 1500 MeV.

III. HIGH-ENERGY BEHAVIOR OF THE NUCLEON SCATTERING CROSS SECTIONS

The high-energy behavior of the nucleon-nucleon, as well as of the nucleon-antinucleon, scattering cross sections is believed to be governed by Regge trajectories representing bound $N\bar{N}$ pairs in the t channel (Fig. 1). In the physical region of the s channel ($s\geqslant 4N^2$, $t<0$), the invariant amplitude $F(s,t)$ describes the scattering process

$$1+2\rightarrow 1'+2',$$

whereas the same function $F(s,t)$, when analytically continued to the physical region of the t channel ($t\geqslant 4N^2$, $s<0$), accounts for the scattering process

$$1+\bar{1}'\rightarrow \bar{2}+2'.$$

Since the object exchanged between 1 and 2 is assumed to possess a definite value for the isobaric spin, we will be interested in such scattering states in the s channel which correspond to a definite isospin state of the pair $1\bar{1}'$ in the t channel. The forward scattering amplitude diagonal in these states may then be written as a sum of Regge poles with a particular value for the isospin.[13,14] The relative signs of the different terms are partly determined by the appropriate Clebsch-Gordan coefficients, and partly by the signature of the trajectories in question. In fact, defining the coupling constant for general values of t as

$$g^2(t)=\beta(t)/\pi\alpha_R'(M^2),$$

and assuming that this expression does not change appreciably and, at least, remains positive definite when extrapolated from $t=M^2$ to $t=0$, we may write

$$\mathrm{Im}F(s,0)=-\sum_r \delta_r\left(\frac{s}{2N^2}\right)^{\alpha_r(0)}\pi g_r^2(0)\epsilon_r, \tag{17}$$

where δ_r is the signature and $\epsilon_r\equiv[(d/dt)\,\mathrm{Re}\alpha(t)]_{t=M^2}$. With these conventions (which seem to differ from those of Ref. 13) we get, remembering that the pseudoscalar trajectories do not contribute at $t=0$, the relations given by Drell.[14] In particular,

[11] S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[12] We use throughout this paper the letter M rather than K^* for the vector meson doublet.

[13] B. M. Udgaonkar, Phys. Rev. Letters 8, 142 (1962).

[14] S. D. Drell, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962).

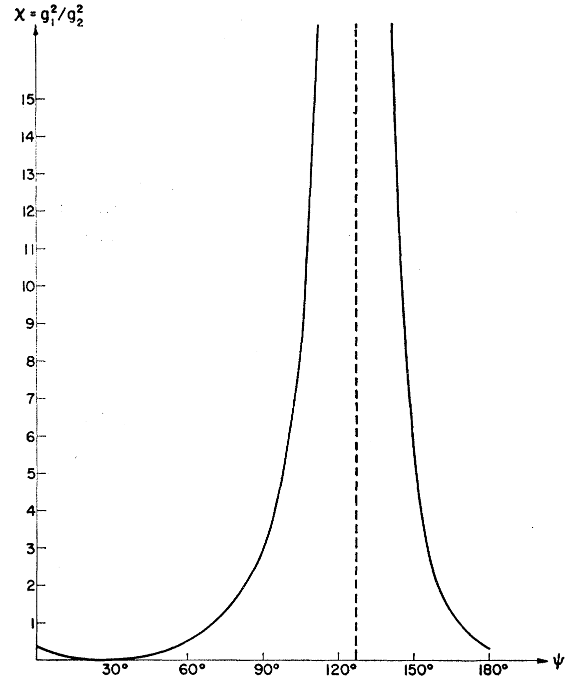

FIG. 4. The ratio $\chi = g_1^2/g_2^2$ calculated from SU_3 symmetry as function of the mixing angle Ψ.

$$
\begin{aligned}
\tfrac{1}{2}(\sigma_{\bar{p}p}-\sigma_{pp}) &= \pi\epsilon_\omega g_{pp\omega}^2(s/2N^2)^{\alpha_\omega-1} \\
&\quad +\pi\epsilon_\varphi g_{pp\varphi}^2(s/2N^2)^{\alpha_\varphi-1} \\
&\quad +\pi\epsilon_\rho g_{pp\rho}^2(s/2N^2)^{\alpha_\rho-1}, \quad (18)\\
\tfrac{1}{2}(\sigma_{np}-\sigma_{pp}) &= \pi\epsilon_\rho g_{pp\rho}^2(s/2N^2)^{\alpha_\rho(0)-1}.
\end{aligned}
$$

Following Drell's analysis we note that in the case that no unitary singlet existed, thus identifying the ω_0 with the $T=0$ member of the octet, the experimental figures for the two differences in Eq. (18) [$\frac{1}{2}(\sigma_{\bar{p}p}-\sigma_{pp})\sim 20$ mb, $\frac{1}{2}(\sigma_{np}-\sigma_{pp})<2$ mb at an energy around 10 BeV] would require

$$g_{pp\omega}^2 \gtrsim 10\, g_{pp\rho}^2, \qquad (19)$$

since the ω and ρ trajectories are not supposed to deviate considerably from each other.

SU_3 symmetry gives the following relation between the two coupling constants (putting $g_{pp\rho}=g_2$):

$$
\begin{aligned}
\chi &= \frac{g_1^2}{g_2^2} = \frac{1}{3}\left(\frac{3a-b}{a+b}\right)^2 \\
&= \frac{(15\sqrt{5})\sin^2\psi + (3\sqrt{5})\cos^2\psi - 30\sin\psi\cos\psi}{(5\sqrt{5})\sin^2\psi + (9\sqrt{5})\cos^2\psi + 30\sin\psi\cos\psi}. \qquad (20)
\end{aligned}
$$

Figure 4 shows this ratio as a function of ψ. The relation (19) restricts the possible values of the mixing angle ψ to the range

$$110° \lesssim \psi \leq 145°. \qquad (21)$$

Although a comparison with Fig. 2 discloses the upper end of the interval (22) to be consistent with a repulsive core in most of the two baryon states, the $(11)_S$ and (30) states are still attractive.

The presence of the singlet invalidates the relation (19). Indeed, if we make the assumption that the ω trajectory, to a reasonably good approximation, coincides with those of the φ and ρ, although it does not belong to the family of octet trajectories, we get instead of (19)

$$h^2 \equiv g^2 + f^2 = g_0^2 + g_1^2 \gtrsim 10\, g_2^2, \qquad (22)$$

where we have used (12). This inequality allows the coupling between the ω and the baryons to be compatible with, or weaker than, the strength of the corresponding ρ coupling. From low-energy S-wave πN scattering and the decay rate of the ρ, Sakurai[15] estimates $g_{\rho pp}^2/4\pi$ to be of the order of 2 or 3. The $g_{\omega pp}^2$ may be determined, for instance, from the leptonic ω decay $\omega \to e^+ + e^-$, or from the two-pion decay branching ratio. It would be very interesting if the ratio χ turned out to be smaller than ten.

IV. INELASTIC NUCLEON-ANTINUCLEON SCATTERING

So far we have only considered processes in which no exchange of hypercharge took place. However, it is possible to gain some additional support for the hypothesis of the existence of the φ by comparing the values of the ratio $g_{pYM}^2/g_{pp\omega}^2$, indirectly obtained from experiments, with the values predicted by SU_3 invariance. For the two diagrams in Fig. 5 we write, respectively,

$$
\begin{aligned}
F_{\rm I}(\vartheta) &= g_{NYM}^2\alpha A(\vartheta), \\
F_{\rm II}(\vartheta) &= g_1^2\beta A(\vartheta), \qquad (23)
\end{aligned}
$$

where α and β are appropriate isospin factors and ϑ is the angle of scattering. At sufficiently high energies we have, if no singlet exists, that

$$\Delta\sigma^{\rm II} \equiv \tfrac{1}{2}[\sigma_{\bar{p}p}-\sigma_{pp}] \approx g_1^2\beta[{\rm Im}A(0)](4\pi/p_c), \quad (24)$$

where p_c is the center-of-mass momentum.

In the expression for the amplitude $F_{\rm I}$ we approximate the angular distribution by an exponential:

$$|A(\vartheta)|^2 = g(\vartheta)|A(0)|^2 \approx e^{\lambda(1-\cos\vartheta)}|A(0)|^2, \quad (25)$$

where the quantities $|A(0)|$ and λ can be determined from a logarithmic plot of the measured angular

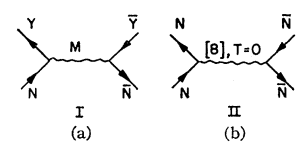

FIG. 5. Vector meson exchange diagrams in $B\bar{B}$ scattering.

[15] J. J. Sakurai, in *Proceedings of the International School of Physics "Enrico Fermi," Varenna*, 1962 (Academic Press Inc., New York, 1962).

distribution. Next, we observe that

$$\frac{\mathrm{Im}A(0)}{|A(0)|}=\frac{-\delta\,\sin\pi\alpha(0)}{|1+\delta e^{-i\pi\alpha(0)}|}$$

$$=\frac{-\delta}{\sqrt{2}}\cdot\frac{\sin\pi\alpha(0)}{[1+\cos\pi\alpha(0)]^{1/2}}\approx\frac{1}{\sqrt{2}},\qquad(26)$$

assuming $\alpha(0)\approx\frac{1}{2}$ and noticing that the signature δ is negative. If the diagram I in Fig. 5 is the dominant one for the production of hyperon-antihyperon pairs in nucleon-antinucleon collisions, we may identify

$$\sigma_{\mathrm{tot}}{}^{I}=\int_{-1}^{1}|A(\cos\vartheta)|^2d(\cos\vartheta).\qquad(27)$$

From (23)–(27) we then get

$$\frac{g_{NYM}{}^2}{g_1{}^2}=\frac{4\pi\lambda^{1/2}}{\sqrt{2}p_c}\frac{\beta}{\alpha}\frac{(\sigma_{\mathrm{tot}}{}^{I})^{1/2}}{\Delta\sigma^{II}}.\qquad(28)$$

Experimental values for the total cross sections, as well as for the angular distributions, are available for the processes

$$\begin{aligned}N+\bar{N}&\to\Lambda+\bar{\Lambda}\\&\to\Sigma^{+}+\bar{\Sigma}^{-}\\&\to\Lambda+\bar{\Sigma}^{0}\\&\to\bar{\Lambda}+\Sigma^{0},\end{aligned}$$

at laboratory momenta of 3 BeV/c,[16] corresponding to a total energy of 2.7 BeV in the center-of-mass system. Unfortunately, this energy is hardly high enough to justify the application of the asymptotic formulas essential for the derivation of Eq. (28). The observed total cross sections were $\sigma_{\Lambda\bar{\Lambda}}=78.5\pm23$ μb, $\sigma_{\Sigma^+\bar{\Sigma}^-}=38\pm7$ μb, $\sigma_{\Lambda\bar{\Sigma}^0}=\sigma_{\Sigma^0\bar{\Lambda}}=45.5\pm13$ μb. The values for the coupling constants are not too sensitive to the precise value of λ. A choice of $\lambda\sim7$ seems to fit the angular distributions reasonably well. Inserting the isospin factors ($\beta=1$, $\alpha=1$, 2, 1 for $\Lambda\bar{\Lambda}$, $\Sigma^+\bar{\Sigma}^-$, and $\Sigma^0\bar{\Lambda}$, respectively), and using the value 15 mb for $\Delta\sigma^{\mathrm{II}}$,[17] we obtain from (28)

$$\begin{aligned}r_1&=g_{p\Lambda M}{}^2/g_1{}^2=0.14,\\r_2&=g_{p\Sigma^+M}{}^2/g_1{}^2=0.05,\\r_3&=r_1/r_2=2.8.\end{aligned}\qquad(29)$$

From SU_3 symmetry the corresponding ratios are given as functions of the mixing parameter:

$$r_1=\frac{(3a+b)^2}{(3a-b)^2}=\frac{5\sin^2\psi+\cos^2\psi+2\sqrt{5}\sin\psi\cos\psi}{5\sin^2\psi+\cos^2\psi-2\sqrt{5}\sin\psi\cos\psi},\qquad(30)$$

$$r_2=\frac{6(b-a)^2}{(3a-b)^2}=\frac{2}{3}\left(\frac{15\sin^2\psi+9\cos^2\psi-6\sqrt{5}\sin\psi\cos\psi}{5\sin^2\psi+\cos^2\psi-2\sqrt{5}\sin\psi\cos\psi}\right).\qquad(31)$$

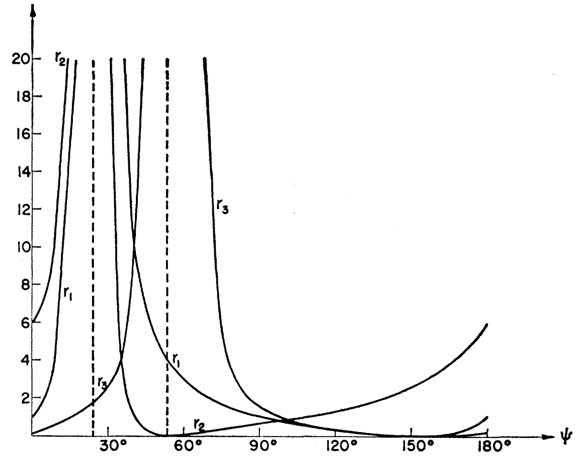

FIG. 6. The ratios r_1, r_2, and r_3 calculated from SU_3 symmetry.

These functions are plotted on Fig. 6, together with the ratio $r_3=g_{p\Lambda M}{}^2/g_{p\Sigma^+M}{}^2$. It is apparent that no choice of the angle ψ can make the predicted values of the ratios (30) and (31) consistent with the values (29). In the presence of the singlet, however, the denominators in (29) are to be replaced by $h^2=g_0{}^2+g_1{}^2$ and, consequently, the numbers quoted there multiplied by $h^2/g_1{}^2$.

It would be tempting to assume that the ratio $r_3=g_{p\Lambda M}{}^2/g_{p\Sigma^+M}{}^2$, as predicted by (30) and (31), is fairly well comparable to the value given by Eq. (29) provided no other graphs involving exchange of other multiplets turn out to be important. Indeed, we might determine the mixing angle ψ by adapting r_3 to the value 2.8 given by (29). The value $\psi=31°$ for the mixing angle thus obtained exhibits a suggestive coincidence with the value $\vartheta=33°$ which is obtained for the mixing angle in the case of baryon-pseudoscalar coupling.[3,18] For $\psi=31°$, (29) is consistent with (30) and (31) if h^2/g^2 is around 400 or

$$g_0{}^2/g_1{}^2\sim400,\qquad(32)$$

which at least agrees with (10). Such large difference in the strength of the two interactions could, of course, hardly be justified if it did concern the actual particles and may be taken as a confirmation of the hypothesis of ω-φ mixing. It may be noticed that the "renormalized" ratio g^2/f^2 is independent of the "bare" ratio $g_0{}^2/f_0{}^2$ is this last number is $\gg1$ [cf. Eq. (12)]. Taking $\psi=31°$, we have

$$g_1{}^2/g_2{}^2\simeq0.02.\qquad(33)$$

The fact that the two mixing angles ψ and ϑ tend

[16] Armentos *et al.*, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962).

[17] G. von Dardel, D. H. Frisch, R. Mermod, R. H. Milburn, P. A. Piroué, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters **5**, 333 (1960). S. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, *ibid.* **7**, 182 (1961).

[18] P. Tarjanne (to be published).

to be of the same order of magnitude makes one suspect that they may be proved to be exactly equal, at least in a certain limit. Indeed, in a self-consistent model, such as that discussed by Cutkosky,[3] the ratio between the mixing angles is in principle fixed by the self-consistency requirement.

In conclusion, the arguments presented above seem to prove that the predictions of SU_3 symmetry are not even qualitatively consistent with the experimental observations if no vector mesons other than the members of the octet are assumed to exist. The inconsistencies can be removed by the introduction of a unitary singlet vector meson for which some empirical support is available. However, it might well be that other more complicated interactions, such as the exchange of members of other multiplets, could play an important role.

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. R. E. Cutkosky who not only suggested the general trend of the present investigation, but also has guided me through the many details of calculation. I should also like to express my gratitude to Dr. P. Tarjanne for what I have learned from him about the group-theoretical aspects of SU_3 symmetry, and to the Carnegie Institute of Technology, Pittsburgh, for granting me the opportunity of joining the Physics department there for a year. Finally, I thank my wife for drawing the curves and for pointing out to me several errors in the numerical calculations.

PHYSICAL REVIEW VOLUME 131, NUMBER 5 1 SEPTEMBER 1963

Analysis of Y_0^* (1520) and Determination of the Σ Parity[*]

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(Received 15 March 1963)

The interaction of K^- mesons on protons resulting in elastic and charge-exchange scattering and hyperon production are reported for a range of momenta from 250 to 513 MeV/c. About 10 000 events obtained in the Lawrence Radiation Laboratory's 15-in. bubble chamber were analyzed. Differential and total cross sections for all channels are examined. For $\Sigma^+\pi^-$, $\Sigma^0\pi^0$, and $\Lambda\pi^0$ production, polarization measurements are also available. A resonant state is identified with a mass 1519 MeV and a full width $\Gamma=16$ MeV, decaying into $\bar{K}N$, $\Sigma\pi$, and $\Lambda\pi\pi$ in the branching ratio 30:55:15, respectively. The resonance is found to have isotopic spin 0 and spin $\frac{3}{2}$, and its parity is that of the $\bar{K}N$ $D_{3/2}$ state. By use of the polarization arising from the $D_{3/2}-S$-wave interference, a strong argument for negative $KN\Sigma$ parity is obtained. All the data are fitted to a model based on a Breit-Wigner resonant amplitude and nonresonant S, P, and D amplitudes, parametrized by constant scattering lengths. An extensive search for χ^2 minima was done on an IBM-7090 computer under various assumptions for the spin and parity of the resonance. Only the $D_{3/2}$ possibility in both K^-p and $\Sigma\pi$ states yields a satisfactory (43% probability) fit. A $D_{5/2}$ K^-p resonance (with $\Sigma\pi$ in $F_{5/2}$) is the nearest alternative possibility, with a likelihood of less than 1% of fitting the data.

I. INTRODUCTION

BELOW 300 MeV/c the K^-p interaction is strongly dominated by S waves. All channels have been found to be satisfactorily described by the S-wave zero-effective-range approximation.[1] Above 300 MeV/c, higher partial waves begin to exhibit themselves in a spectacular fashion. Previous to this experiment, an analysis of 140 interactions at 400 MeV/c in the 15-in. liquid-hydrogen bubble chamber indicated a large $\cos^2\theta$ term in the elastic angular distribution.[2] With this guidance, a much more detailed study of this region was begun in 1960. Over 10 000 events have been analyzed at K^- laboratory momenta of about 300, 350, 400, 440, and 510 MeV/c and form the source of the data presented here. Apart from the addition of 1000 more K^-p elastic scatters and the inclusion of a "beam averaging" procedure, the data are essentially the same as reported in preliminary form earlier.[3] Computer fits to the final data presented here yield resonance parameters very similar to those found in the precomputer analysis.

In Sec. II we discuss the K^- beam, and in Sec. III, the scanning and measuring procedure. The results of the measurements and remarks on experimental biases appear in Sec. IV. A simplified discussion of the resonance is found in Secs. V and VI, where we establish the properties of the resonance and develop the argument

* This work was done under the auspices of the U. S. Atomic Energy Commission.
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